The Universal Progressive Income Tax Equation

Peter Rowntree 1996, 2005

A method of computing income tax using an integral of the Sigmoid Function.

Some definitions:

$r_t \in [0,1]$ = the top tax rate

$r_b \in [0,1]$ = the bottom tax rate

where:

$r_b \le r_t$

$r_e \in [-\infty,1]$ = the effective tax rate

$i \in (0,\infty]$ = the socially ideal income

$m \in [0,\infty]$ = the minimum income

$x \in [0,\infty]$ = the taxpayer's income

$z \in [-\infty,0)$ = the "zero point"

= the value in the domain of Eq. (2) corresponding to 0 in the income domain

= the "steepness" of the S-curve

where:

r_t, r_b, i, m and z are parameters set by the taxing entity (a national government, for example).

Now, we map x into the domain of the Sigmoid Function:

$$y = z - (z/i)x \qquad (1)$$

and:

$$S(y) = \frac{1}{1+e^{-y}} \qquad \text{(the Sigmoid Function)} \quad (2)$$

and:

$$A(y) = \int S(y)dy = \log(1+e^{y}) + C \qquad (3)$$

where:

$$C = -\log(1+e^{z})$$

and, since it, along with C, only changes when taxing entity parameters change, let's define a factor to map $A(y)$ back into "income space":

$$f = (r_t - r_b)(i/-z)$$

which yields the Universal Progressive Income Tax Equation:

$$T(x) = \left(\log(1+e^{y}) + C\right) f + r_b x - m \qquad (4)$$

Some characteristics:

$r_e = T(x)/x$, for $x \ne 0$

$\lim_{z \to -\infty} S$ = a step function

$\lim_{z \to 0} S(y) = 0.5$

$\lim_{r_b \to r_t} T$ = the Flat Tax, often pitched to the poor by the Disingenuous Rich as "tax code simplification".

A nice additional consequence of continuous-function progressive taxation is the flattening of the business cycle; any drop in aggregate demand is automatically corrected by perfectly targeted tax relief for the affected consumers.